process relation [Eq. (4)]† in place of the tangent-gas law [Eq. (9)] brings in additional nonlinearity that has trivial consequences, if compared to the nonlinear interaction between pressure waves and the combustion processes. Although the equations become less transparent, conclusions derived from the mathematical model are not modified essentially if Eq. (9) is replaced by a polytropic process relation.‡ Therefore, in order to make new and interesting phenomena clearly evident, convective nonlinearity is suppressed in the theoretical treatment[1] by evoking the tangent-gas law. Emphasizing the shortcomings of a burning rate function like Eq. (8), a representation of the combustion mechanism which takes account of vaporization and chemical reaction but cannot take account of physical mixing and molecular diffusion (processes that are rate-controlling in many combustion fields) is earnest criticism. By introducing a time delay function in place of Eq. (8), by linearizing the system of equations, and by supplementing the equations that result with boundary conditions, it is possible to obtain a resonance-type mechanism for instability, a mechanism that does not appear in the idealized model of Ref. 1. Yet the issue is not whether a representation of the combustion mechanism like Eq. (8) is of completely general practical interest, but rather whether it is generally less appropriate than the ad hoc introduction of a time delay function. It certainly is interesting that a mechanism for instability exists even if processes associated with a time delay (such as physical mixing and molecular diffusion) are not rate-controlling. In fact, this "friction term" interaction between pressure waves and the combustion processes must appear in an all-embracing mathematical theory, one with additional equations for mixing and diffusion, because the idealized mathematical model in Ref. 1 is the limit of such a theory with "time delays" permitted to vanish. Furthermore, it is questionable whether a resonance-type mechanism for instability can be derived in rigorous fashion from the complete set of governing equations in an all-embracing mathematical theory.§ No one can argue that the complicated and variegated combustion fields of practical interest are not worthy of analysis from complementary points of view.

For practical situations that are described approximately by the idealized model, consider whether the mechanism for instability is essentially local in character, as asserted in Ref. 1. Provided that it is appropriate to relate the idealized mathematical model to an actual combustion field, Eq. (19)

$$\frac{1}{2}(1+\omega_0)a\frac{\partial^2 P}{\partial t^2} + \bar{\phi}[a(m+1)P^{m-1} - b(m-1)P^{m-2}] \times \frac{\partial P}{\partial t} - \frac{\partial^2 P}{\partial \psi^2} = 0$$

governs the dynamics of pressure waves in the region of active combustion. Observe that the equation has the deflagration solution $P \equiv P_c =$ const, with P_c interpreted physically as the average "chamber pressure" for steady, normal combustion. In the neighborhood of the $P \equiv P_c$ solution, the equation has the general acoustical perturbative solution

$$\frac{P}{P_c} = 1 + \sum_n \epsilon_n e^{-\mu_n t}\sin(\nu_n t + \delta_n) \times \sin\left(k_n\left[\frac{(1+\omega_0)a}{2}\right]^{1/2}\psi + \theta_n\right)$$

where the ϵ's are arbitrary small constants, $\epsilon_n^2 \ll 1$, the k's denote wave-numbers in ψ space for acoustical disturbances that are consistent with conditions at a remote boundary (well outside the region of active combustion), the δ's and θ's denote phase constants that are prescribed by the same boundary conditions, and the μ's and ν's are given by

$$\left.\begin{aligned}\mu_n &= \chi \pm (\chi^2 - k_n^2)^{1/2} \\ \nu_n &= 0\end{aligned}\right\} \quad [k_n^2 \leqslant \chi^2]$$

$$\left.\begin{aligned}\mu_n &= \chi \\ \nu_n &= \pm (k_n^2 - \chi^2)^{1/2}\end{aligned}\right\} \quad [k_n^2 \geqslant \chi^2]$$

with the abbreviation

$$\chi \equiv \frac{\bar{\phi}[a(m+1)P_c^{m-1} - b(m-1)P_c^{m-2}]}{(1+\omega_0)a}$$

Clearly, the $P \equiv P_c$ solution is stable if all of the μ's are positive and unstable if any μ_n is negative. But, independent of the magnitude of each admissible wave number k_n (or phase constants δ_n and θ_n), each μ_n is positive (negative) if the parameter χ is positive (negative). Thus the stability depends entirely on the sign of χ and does not depend on boundary conditions that fix the k's, the δ's, and the θ's. Hence the mechanism for instability is essentially local in character. The condition for stability, namely, $\chi > 0$, is recast in a neat form by introducing the effective polytropic index, as by Eq. (27):

$$\kappa = (b/aP_c) - 1$$

Then the necessary and sufficient condition for pressure wave stability is obtained as‖

$$m < [1 + (2/\kappa)]$$

a result derived by alternative considerations and reported in Ref. 1.

References

[1] Rosen, G., "Stability of pressure waves in a combustion field," ARS J. **32**, 1605–1607 (1962).

[2] Rosen, G., "Nonlinear pressure oscillations in a combustion field," ARS J. **30**, 422–423 (1960).

‖ Note that, if this condition for pressure wave stability is satisfied, it is satisfied by only a slim margin, since m ranges between 1 and 3 in practical cases, while $[1 + (2/\kappa)]$ ranges between 2 and 3 for real gases. For larger values of the "chamber pressure" P_c, pressure wave stability is less likely; the rate-controlling physical and/or chemical processes usually are associated with a larger value for m at higher pressures, binary and ternary molecular processes playing a more significant role.

† Equations quoted are in Ref. 1.

‡ For example, the necessary and sufficient condition for stable pressure waves $\{m < [1 + (2/\kappa)]\}$ still is obtained if one replaces Eq. (9) with a polytropic process relation, as one may deduce easily from the results in Ref. 2.

§ Attempts by the author to obtain theoretical results of this nature were not fruitful.

Comments on "Free Vibration of a Damped Elliptical Plate"

D. J. Johns*

College of Aeronautics, Cranfield, England

In Ref. 1, results are presented for the two lowest eigenfrequencies of a clamped-edge elliptical plate. It should be pointed out, however, that only one of the eigenfrequencies determined has any physical significance, viz., the solution corresponding to λ_1^2, since the assumed modal form ϕ can-

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* Lecturer, Department of Aircraft Design.

not be considered to represent any mode other than the fundamental vibration mode. Using

$$\phi = A_1\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)^2 + A_2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)^3 \quad (1)$$

the results quoted for λ_1^2 and λ_2^2 were

$$\lambda_1^2 = 1.6341(24/a^4)[1 + (2a^2/3b^2) + (a^4/b^4)] \quad (2)$$

and

$$\lambda_2^2 = 5.3825(24/a^4)[1 + (2a^2/3b^2) + (a^4/b^4)] \quad (3)$$

In Ref. 2, the present writer assumed the following modal form in a panel flutter analysis:

$$\phi = A_1\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)^2 + A_2x\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)^2 \quad (4)$$

and the first two eigenfrequencies found for these two uncoupled modes are expressed by

$$\lambda_1^2 = (40/a^4)[1 + (2a^2/3b^2) + (a^4/b^4)] \quad (5)$$

and

$$\lambda_2^2 = (300/a^4)[1 + (2a^2/5b^2) + (a^4/5b^4)] \quad (6)$$

It can be deduced by comparison of Eqs. (2) and (5) that the inclusion of the additional A_2 term in Eq. (1) has had very little influence on the value of the lowest eigenfrequency determined.

To assess the accuracy of the forementioned approximate results, comparison can be made for the particular case of a clamped-edge circular plate with the exact results presented in Ref. 3 for the first two eigenfrequencies, i.e.,

$$\lambda_1^2 = 105/a^4 \qquad \lambda_2^2 = 464/a^4 \quad (7)$$

The corresponding results from Eqs. (5) and (6) are

$$\lambda_1^2 = 106.67/a^4 \qquad \lambda_2^2 = 480/a^4 \quad (8)$$

whereas from Eqs. (2) and (3), one obtains

$$\lambda_1^2 = 104.58/a^4 \qquad \lambda_2^2 = 344/a^4 \quad (9)$$

Thus, although the two-term analysis of Ref. 1 gives a better approximate result for the lowest eigenfrequency than a one-term analysis, the expression quoted in Ref. 1 for the second eigenfrequency has no physical significance.

References

[1] McNitt, R. P., "Free vibration of a damped elliptical plate," J. Aerospace Sci. **29** (1962).

[2] Johns, D. J., "Some panel flutter studies using piston theory," J. Aerospace Sci. **25** (1958).

[3] Rattaya, J. V., "Flutter analysis of circular panels" J., Aerospace Sci. **29**, 578–582 (1962).

Book Notes

Space Research and Technology, edited by G. V. E. Thompson (Gordon and Breach Science Publishers, New York, 1962), 216 pp. $5.95.

Contents: 31 papers contributed by different authors and divided into 5 major sections. Section 1) Space Medicine Symposium; Section 2) Rocket and Satellite Instrumentation Symposium; Section 3) High Altitude Chambers and Pressure Suits and Their Part in Manned Flight to the Moon; Section 4) Space Navigation Symposium; Section 5) Liquid Hydrogen Symposium.

This volume contains the Proceedings of four Symposia sponsored by the British Interplanetary Society. Engineers and scientists from five different countries, including the United States, have papers included in the volume.

Developments in Theoretical and Applied Mechanics, edited by the Technical Publications Department, Technical Information Division, Oak Ridge National Laboratory (Plenum Press, New York, 1963), Vol. 1, 519 pp. $16.00.

Contents: 32 papers contributed by different authors and divided into 4 major parts. Part 1) Solid Mechanics; Part 2) Fluid Mechanics; Part 3) Dynamics and Vibrations; Part 4) Experimental and Applied Mechanics.

This volume contains the Proceedings of the First Southeastern Conference on Theoretical and Applied Mechanics, held at Gatlinburg, Tenn., May 3–4, 1962, and sponsored by the Oak Ridge National Laboratory. Engineers and physicists will find in this book material both valuable in the solution of practical problems and useful as suggestions for theoretical treatments of the many aspects of mechanics.

Electric Circuit Analogies for Elastic Structures, Richard H. MacNeal, *Computer Engineering Associates, Pasadena, Calif.* (John Wiley & Sons Inc., New York, 1962), Vol. 2, 262 pp. $11.50.

Chapters: 1) Introduction; 2) Electrical Circuit Analysis; 3) Basic Techniques Used in Deriving Analogies; 4) Analogies for Structures That Consist of One-Dimensional Elements; 5) Analogies for Two-Dimensional Structural Elements; 6) Practical Considerations in the Construction of Analogies for Static Structural Analysis; 7) Examples of Aircraft Static Structural Analysis; 8) Dynamic Analysis; 9) Applications to Other Types of Structural Analysis; 10) General Theorems and Special Synthesis Procedures.

This book indicates and exploits the similarities between concepts in electric circuit theory and concepts in elastic structure theory. Specialized knowledge of electric circuit theory is not a prerequisite, since the required circuit theory principles are developed in the book. This volume may be used as an introductory text for those wishing knowledge of the analogies between mechanics and electricity, as a compilation of methods and techniques for users of direct analog computing, and as an exposition of the scope of direct analog computer methods in the solution of structural problems.

Navigation and Guidance in Space, Edward V. B. Stearns, *Manager, Advanced Program Development, Space Systems Division, Lockheed Missiles and Space Company* (Prentice-Hall Inc., Englewood Cliffs, N. J., 1963), 341 pp. $12.00.

Chapters: 1) Introduction to Space Flight; 2) Orbit Characteristics; 3) Instrumentation; 4) Guidance of the Ballistic Missile; 5) Navigation for Satellites; 6) Interplanetary Navigation; 7) Navigation to the Moon; 8) Navigation in Interstellar Flight. *Appendixes:* 1) Glossary of Terms; 2) Glossary of Symbols; 3) Data on the Solar System; 4) Relation between Coordinates and Parameters; 5) Orbit Equations and Associated Relations; 6) Conversion Tables.

This book treats the problem of space navigation in each of the several environments. Emphasis is placed on navigation rather than on guidance systems design or trajectory calculations. The bulk of the text material deals with spacecraft guidance as though the function actually were performed by a crew member.

Developments in Mechanics, edited by J. E. Lay, *Professor of Mechanical Engineering, Michigan State University,* and L. E. Malvern, *Professor of Applied Mechanics, Michigan State University* (Plenum Press,

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The books listed here are those recently received by the AIAA from various publishers who wish to announce their current offerings in the field of astronautics. The order of listings does not necessarily indicate the editors' opinion of their relative importance or competence.